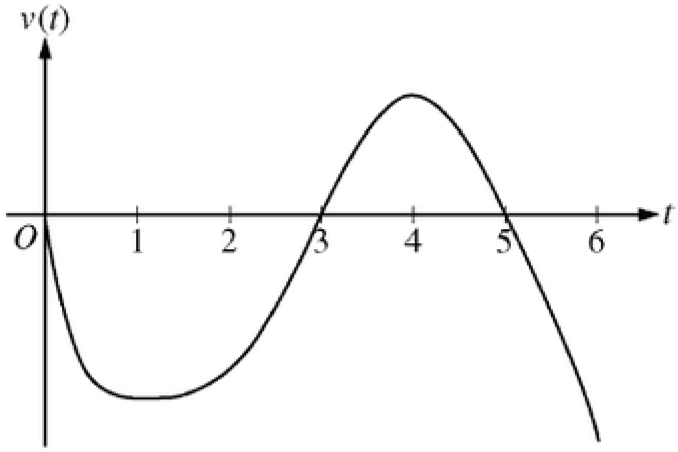
k Appl Dérivées I - Q2



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$,$t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the x-axis and the graph of v on the intervals [0,3] ,[3,5] and [5,6] are 8, 3, and 2 respectively. At time $t = 0$, the particle is $x = -2$.

1. For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

Part B

3 points can be earned.

1 point for positions at $t=3$, $t=5$ and $t=6$.

1 point for description of motion.

1 point for conclusion.

The particle moves continuously and monotonically from $x(0)=-2$ to $x(3)=-10$. Similarly, the particle moves continuously and monotonically from $x(3)=-10$ to $x(5)=-7$ and also from $x(5)=-7$

k Appl Dérivées I - Q2

to $x(6)=-9$.

0	1	2	3

3 points can be earned.

1 point for positions at $t=3$, $t=5$ and $t=6$.

1 point for description of motion.

1 point for conclusion.

The particle moves continuously and monotonically from $x(0)=-2$ to $x(3)=-10$. Similarly, the particle moves continuously and monotonically from $x(3)=-10$ to $x(5)=-7$ and also from $x(5)=-7$ to $x(6)=-9$.

---

2. Let f be a function that is continuous on the closed interval [1, 3] with f(1) = 10 and f(3) = 18. Which of the following statements must be true?

(A) $10 \leq f(2) \leq 18$

(B) f is increasing on the interval [1, 3]

(C) f(x) = 17 has at least one solution in the interval [1, 3] ✔

(D) f' (x) = 8 has at least one solution in the interval (1, 3).

(E) $\int_1^3 f(x)dx > 20$

---

k Appl Dérivées I - Q2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon (t = 0) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

3. For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

Part C

The response can earn up to 3 points:

1 point: For considering change in sign of L'

1 point: For the analysis

1 point: For the correct conclusion

L is differentiable [0,9] so the Mean Value Theorem implies $L'(t) > 0$ for some t in (1,3) and some t in (4,7). Similarly, $L'(t) < 0$ for some t in (3,4) and some t in (7,8). Then, since L' is continuous on [0,9], the Intermediate Value Theorem implies that $L'(t)=0$ for at least three values of t in [0,9].

Alternate solution:

1 point: Considers relative extrema of L on (0,9)

1 point: For the analysis

1 point: For the correct conclusion

The continuity of L on [1,4] implies that L attains a maximum value there. Since L(3) > L(1) and L(3) > L(4), this maximum occurs on (1,4). Similarly, L attains a minimum on (3,7) and a maximum on (4,8). L is differentiable, so L'(t) = 0 at each relative extreme point on (0,9). Therefore L'(t) = 0 for at least three values of t in [0,9].

k Appl Dérivées I - Q2

[Note: There is a function L that satisfies the given conditions with L'(t)=0 for exactly three values of t]

0	1	2	3

The response earns all three of the following points:

1 point: For considering change in sign of L'

1 point: For the analysis

1 point: For the correct conclusion

L is differentiable [0,9] so the Mean Value Theorem implies $L'(t) > 0$ for some t in (1,3) and some t in (4,7). Similarly, $L'(t) < 0$ for some t in (3,4) and some t in (7,8). Then, since L' is continuous on [0,9], the Intermediate Value Theorem implies that $L'(t)=0$ for at least three values of t in [0,9].

Alternate solution:

1 point: Considers relative extrema of L on (0,9)

1 point: For the analysis

1 point: For the correct conclusion

The continuity of L on [1,4] implies that L attains a maximum value there. Since L(3) > L(1) and L(3) > L(4), this maximum occurs on (1,4). Similarly, L attains a minimum on (3,7) and a maximum on (4,8). L is differentiable, so L'(t) = 0 at each relative extreme point on (0,9). Therefore L'(t) = 0 for at least three values of t in [0,9].

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	−120	−150

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes.

k Appl Dérivées I - Q2

Selected values for $v_A(t)$ are given in the table above.

4. Do the data in the table support the conclusion that train *A*'s velocity is -100 meters per minute at some time *t* with 5 < *t* < 8 ? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for $V_A(8)$ < -100 < $V_A(5)$

1 point is earned for the conclusion, using IVT

V_A is differentiable ⇒ V_A is continuous

$V_A(8)$ = -120 < -100 < 40 = $V_A(5)$

Therefore, by the Intermediate Value Theorem, there is a time *t*, 5<*t*<8, such that V_A (t) = -100.

0	1	2

The student response earns two of the following points:

1 point is earned for $V_A(8)$ < -100 < $V_A(5)$

1 point is earned for the conclusion, using IVT

V_A is differentiable ⇒ V_A is continuous

$V_A(8)$ = -120 < -100 < 40 = $V_A(5)$ Therefore, by the Intermediate Value Theorem, there is a time *t*, 5<*t*<8, such that V_A (t) = -100.

k Appl Dérivées I - Q2

5. Let *f* be a function that is continuous on the closed interval $[2,\ 4]$ with $f\,(2) = 10$ and $f\,(4) = 20.$ Which of the following is guaranteed by the Intermediate Value Theorem?

(A) $f\,(x) = 13$ has at least one solution in the open interval $(2,\ 4)$. ✔

(B) $f\,(3) = 15$

(C) *f* attains a maximum on the open interval $(2,\ 4)$.

(D) $f'\,(x) = 5$ has at least one solution in the open interval $(2,\ 4)$.

(E) $f'\,(x) > 0$ for all *x* in the open interval $(2,\ 4)$.

---

6. A polynomial *p(x)* has a relative maximum at (-2,4) , a relative minimum at (1,1) , a relative maximum at (5, 7) and no other critical points. How many zeros does p(x) have?

(A) One

(B) Two ✔

(C) Three

(D) Four

(E) Five

---

7. $f\,(x) = \frac{x^4+x^3+x^2+x+1}{380-\ln\left(\frac{x^2+1}{2}\right)}$

Let f be the function defined above. The Intermediate Value Theorem applied to f on the closed interval $[10, 12]$ guarantees a solution in $[10, 12]$ to which of the following equations?

k Appl Dérivées I - Q2

(A) $f(x) = 0$

(B) $f(x) = 27.372$

(C) $f(x) = 42.421$ ✔

(D) $f(x) = 67.205$

8.

x	-2	-1	0	1	2	3
$f(x)$	-2	5	2	-4	-1	3

Selected values of a continuous function f are given in the table above. What is the fewest possible number of zeros of f in the interval $[-2, 3]$?

(A) Zero

(B) One

(C) Two

(D) Three ✔

---

9. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly

k Appl Dérivées I - Q2

label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

t (hours)	0	3	8	12	14
$S'(t)$ (centimeters per hour)	2.3	2.1	1.9	1.7	1.6

The depth of snow in a field is given by the twice-differentiable function S for $0 \leq t \leq 14$, where $S(t)$ is measured in centimeters and time t is measured in hours. Values of $S'(t)$, the derivative of S, at selected values of time t are shown in the table above. It is known that the graph of S is concave down for $0 \leq t \leq 14$.

(a) Use the data in the table to approximate $S''(10)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $S''(10)$ in the context of the problem.

Please respond on separate paper, following directions from your teacher.

(b) Is there a time t, for $0 \leq t \leq 14$, at which the depth of snow is changing at a rate of 2 centimeters per hour? Justify your answer.

Please respond on separate paper, following directions from your teacher.

(c) At time $t = 8$, the depth of snow is 45 centimeters. Use the line tangent to the graph of S at $t = 8$ to approximate the depth of snow at time $t = 10$. Is the approximation an underestimate or an overestimate of the actual depth of snow at time $t = 10$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

(d) The depth of snow, in centimeters, is also modeled by the twice-differentiable function D for $0 \leq t \leq 14$, where $D(t) = 120 - 92e^{-\frac{t}{40}}$ and time t is measured in hours. Use the model to find $D'(10)$. Using correct units, explain the meaning of $D'(10)$ in the context of the problem.

Please respond on separate paper, following directions from your teacher.

Part A

The first point requires a correct substitution of values. A simplified answer is not required.

The second point requires a time reference, rate, and units. The point may be earned with an incorrect approximation based on one computational error and provided the behavior described is consistent with the sign of the approximation, the numerical value, and the time reference.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ approximation
- ☐ interpretation with units

Solution:

$S''(10) \approx \frac{S'(12) - S'(8)}{12 - 8} = \frac{1.7 - 1.9}{4} = -0.05$

At time $t = 10$ hours, the rate of change of the depth of snow is changing at a rate of -0.05 centimeter per hour per hour.

Part B

The second point may be earned if response contains conditions of IVT without naming the theorem.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ S' is continuous
- ☐ justification using Intermediate Value Theorem

Solution:

S is twice differentiable. $\Rightarrow$ S' is differentiable and thus continuous.

$S'(8) = 1.9 < 2 < 2.1 = S'(3)$

By the Intermediate Value Theorem, there is a value of t, for $0 \leq t \leq 14$, such that $S'(t) = 2$.

Part C

The second point may be earned if using an incorrect value for slope, provided the value is labeled as $S'(8)$.

The second point does not require a simplified answer. Substitution of function values is required.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3

k Appl Dérivées I - Q2

The student response accurately includes all three of the criteria below.

- ☐ equation for tangent line
- ☐ approximation
- ☐ overestimate with justification using concavity

Solution:

$S'(8) = 1.9$ from table

An equation for the tangent line is $y = 45 + 1.9(t - 8)$.

$S(10) \approx 45 + S'(8)(10 - 8) = 48.8$ centimeters

This approximation is an overestimate because the graph of S is concave down for $8 \leq t \leq 10$.

Part D

The second point requires a time reference, rate, and units. The point may be earned with an incorrect approximation based on one computational error and provided the behavior described is consistent with the sign of the approximation, the numerical value, and the time reference.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ $D'(10)$
- ☐ interpretation with units

Solution:

$D'(10) = 1.791$

k Appl Dérivées I - Q2

At time $t = 10$ hours, the depth of snow is changing by 1.791 centimeters per hour.