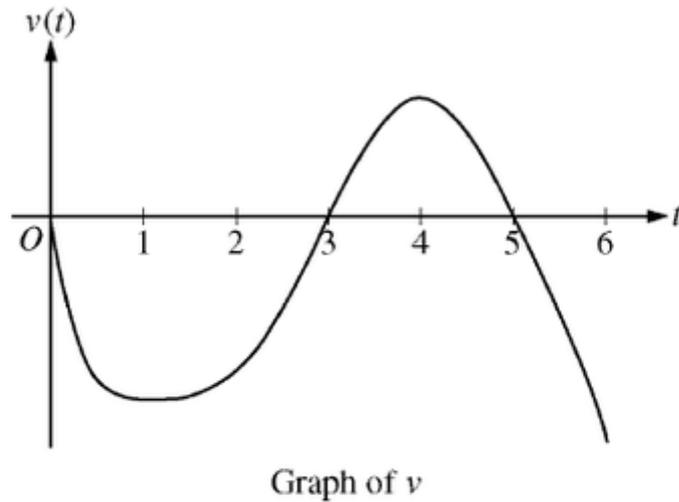


## k Appl Dérivées I - Q2

Name \_\_\_\_\_



A particle moves along the  $x$ -axis so that its velocity at time  $t$ , for  $0 \leq t \leq 6$ , is given by a differentiable function  $v$  whose graph is shown above. The velocity is 0 at  $t = 0$ ,  $t = 3$ , and  $t = 5$ , and the graph has horizontal tangents at  $t = 1$  and  $t = 4$ . The areas of the regions bounded by the  $x$ -axis and the graph of  $v$  on the intervals  $[0,3]$ ,  $[3,5]$  and  $[5,6]$  are 8, 3, and 2 respectively. At time  $t = 0$ , the particle is  $x = -2$ .

1. For how many values of  $t$ , where  $0 \leq t \leq 6$ , is the particle at  $x = -8$ ? Explain your reasoning.



Please respond on separate paper, following directions from your teacher.

2. Let  $f$  be a function that is continuous on the closed interval  $[1, 3]$  with  $f(1) = 10$  and  $f(3) = 18$ . Which of the following statements must be true?



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- (A)  $10 \leq f(2) \leq 18$
- (B)  $f$  is increasing on the interval  $[1, 3]$
- (C)  $f(x) = 17$  has at least one solution in the interval  $[1, 3]$
- (D)  $f'(x) = 8$  has at least one solution in the interval  $(1, 3)$ .
- (E)  $\int_1^3 f(x)dx > 20$
- 

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.

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3. For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.



Please respond on separate paper, following directions from your teacher.

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$t$ (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in



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meters per minute, is given by a differentiable function  $v_A(t)$ , where time  $t$  is measured in minutes. Selected values for  $v_A(t)$  are given in the table above.

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4. Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time  $t$  with  $5 < t < 8$ ? Give a reason for your answer.



Please respond on separate paper, following directions from your teacher.

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5. Let  $f$  be a function that is continuous on the closed interval  $[2, 4]$  with  $f(2) = 10$  and  $f(4) = 20$ . Which of the following is guaranteed by the Intermediate Value Theorem?

- (A)  $f(x) = 13$  has at least one solution in the open interval  $(2, 4)$ .
- (B)  $f(3) = 15$
- (C)  $f$  attains a maximum on the open interval  $(2, 4)$ .
- (D)  $f'(x) = 5$  has at least one solution in the open interval  $(2, 4)$ .
- (E)  $f'(x) > 0$  for all  $x$  in the open interval  $(2, 4)$ .
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6. A polynomial  $p(x)$  has a relative maximum at  $(-2, 4)$ , a relative minimum at  $(1, 1)$ , a relative maximum at  $(5, 7)$  and no other critical points. How many zeros does  $p(x)$  have?



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- (A) One
- (B) Two
- (C) Three
- (D) Four
- (E) Five
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7. 
$$f(x) = \frac{x^4 + x^3 + x^2 + x + 1}{380 - \ln\left(\frac{x^2 + 1}{2}\right)}$$

Let  $f$  be the function defined above. The Intermediate Value Theorem applied to  $f$  on the closed interval  $[10, 12]$  guarantees a solution in  $[10, 12]$  to which of the following equations?

- (A)  $f(x) = 0$
- (B)  $f(x) = 27.372$
- (C)  $f(x) = 42.421$
- (D)  $f(x) = 67.205$
- 

8.

$x$	-2	-1	0	1	2	3
$f(x)$	-2	5	2	-4	-1	3

Selected values of a continuous function  $f$  are given in the table above. What is the fewest possible number of zeros of  $f$  in the interval  $[-2, 3]$ ?



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- (A) Zero
- (B) One
- (C) Two
- (D) Three
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**9. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.**

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

$t$ (hours)	0	3	8	12	14
$S'(t)$ (centimeters per hour)	2.3	2.1	1.9	1.7	1.6

The depth of snow in a field is given by the twice-differentiable function  $S$  for  $0 \leq t \leq 14$ , where



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$S(t)$  is measured in centimeters and time  $t$  is measured in hours. Values of  $S'(t)$ , the derivative of  $S$ , at selected values of time  $t$  are shown in the table above. It is known that the graph of  $S$  is concave down for  $0 \leq t \leq 14$ .

(a) Use the data in the table to approximate  $S''(10)$ . Show the computations that lead to your answer. Using correct units, explain the meaning of  $S''(10)$  in the context of the problem.



Please respond on separate paper, following directions from your teacher.

(b) Is there a time  $t$ , for  $0 \leq t \leq 14$ , at which the depth of snow is changing at a rate of 2 centimeters per hour? Justify your answer.



Please respond on separate paper, following directions from your teacher.

(c) At time  $t = 8$ , the depth of snow is 45 centimeters. Use the line tangent to the graph of  $S$  at  $t = 8$  to approximate the depth of snow at time  $t = 10$ . Is the approximation an underestimate or an overestimate of the actual depth of snow at time  $t = 10$ ? Justify your answer.



Please respond on separate paper, following directions from your teacher.

(d) The depth of snow, in centimeters, is also modeled by the twice-differentiable function  $D$  for  $0 \leq t \leq 14$ , where  $D(t) = 120 - 92e^{-\frac{t}{40}}$  and time  $t$  is measured in hours. Use the model to find  $D'(10)$ . Using correct units, explain the meaning of  $D'(10)$  in the context of the problem.



Please respond on separate paper, following directions from your teacher.

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