Big Idea 1: Limits

	Enduring Understandings (Students will understand that…)	Learning Objectives (Students will be able to…)	Essential Knowledge Statements (Students will know that…)
Big Idea #1 Limits	EU 1.1: The concept of a limit can be used to understand the behavior of function.	LO 1.1A(a): Express limits symbolically using correct notation. LO 1.1A(b): Interpret limits expressed symbolically.	EK 1.1A1: Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim\limits_{x\to c} f(x) = R$.
			Exclusion Statement (EK 1.1A1): *The epsilon delta definition of a limit is not assessed on the AP Calculus AB or BC Exam. Teachers, however, may include this topic in the course if time permits.*
			EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
			EK 1.1A3: A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or of the limit from the left does not equal the limit from the right.
			Examples of limits that do not exist: $\lim\limits_{x\to 0} \frac{1}{x^2} = \infty$ $\lim\limits_{x\to 0} \sin\left(\frac{1}{x}\right)$ *does not exist* $\lim\limits_{x\to 0} \frac{\lvert x\rvert}{x}$ *does not exist* $\lim\limits_{x\to 0} \frac{1}{x}$ *does not exist*
		LO 1.1B: Estimate the limits of functions.	EK 1.1B1: Numerical and graphical information can be used to estimate limits.
		LO 1.1C: Determine limits of functions.	EK 1.1C1: Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
			EK 1.1C2: The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
			EK 1.1C3: Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hôpital's Rule.
		LO 1.1D:	EK 1.1D1: Asymptotic and unbounded behavior of functions can be explained and described using limits.
			EK 1.1D2: Relative magnitudes of functions and their rates of change can be compared using limits.
	EU 1.2: Continuity is a key property of functions that is defined using limits.	LO 1.2A: Analyze functions for intervals of continuity or points of discontinuity.	EK 1.2A1: A function is continuous at $x = c$ provided that $f(c)$ exists, $\lim\limits_{x\to c} f(x)$ exists, and $\lim\limits_{x\to c} f(x) = f(c)$.
			EK 1.2A2: Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
			EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
		LO 1.2B: Determine the applicability of important calculus theorems using continuity.	EK 1.2B1: Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.

Big Idea 2: Derivatives

	Enduring Understandings (Students will understand that…)	Learning Objectives (Students will be able to…)	Essential Knowledge Statements (Students will know that…)
Big Idea #2 Derivatives	EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.	LO 2.1A: Identify the derivative of a function as the limit of a difference quotient.	EK 2.1A1: The difference quotients $\dfrac{f(a+h)-f(a)}{h}$ and $\dfrac{f(x)-f(a)}{x-a}$ express the average rate of change over an interval.
			EK 2.1A2: The instantaneous rate of change of a function at a point can be expressed by $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ or $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$, provided the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
			EK 2.1A3: The derivative of f is the function whose value at x is $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ provided this limit exists.
			EK 2.1A4: For $y=f(x)$, notations for the derivative include $\dfrac{dy}{dx}$, $f'(x)$, and y'.
			EK 2.1A5: The derivative can be represented graphically, numerically, analytically, and verbally.
		LO 2.1B: Estimate derivatives.	EK 2.1B1: The derivative at a point can be estimated from information given in tables or graphs.
		LO 2.1C: Calculate derivatives.	EK 2.1C1: Direct application of the definition of the derivative can be used to find the derivative of selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
			EK 2.1C2: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
			EK 2.1C3: Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
			EK 2.1C4: The chain rule provides a way to differentiate composite functions.
			EK 2.1C5: The chain rule is the basis for implicit differentiation.
			EK 2.1C6: The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
		LO 2.1D: Determine higher ordered derivatives.	EK 2.1D1: Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.
			EK 2.1D2: Higher order derivatives are represented by a variety of notations. For $y=f(x)$, notations for the second derivative include $\dfrac{d^2y}{dx^2}$, $f''(x)$, and y''. Higher order derivatives can be denoted $\dfrac{d^ny}{dx^n}$ or $f^{(n)}(x)$.

Big Idea 2: Derivatives (continued)

	Enduring Understandings (Students will understand that…)	Learning Objectives (Students will be able to…)	Essential Knowledge Statements (Students will know that…)
Big Idea #2 Derivatives	EU 2.2: A function's derivative, which itself a function, can be used to understand the behavior of the function.	LO 2.2A: Use derivatives to analyze properties of a function.	EK 2.2A1: First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
			EK 2.2A2: Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
			EK 2.2A3: Key features of the graphs of f, f', and f'' are related to one another.
		LO 2.2B: Recognize the connection between differentiability and continuity.	EK 2.2B1: A continuous function may fail to be differentiable at a point in its domain.
			EK 2.2B2: If a function is differentiable at a point, then it is continuous at that point.
	EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.	LO 2.3A: Interpret the meaning of a derivative within a problem.	EK 2.3A1: The unit for $f'(x)$ is the unit for f divided by the unit for x.
			EK 2.3A2: The derivative of a function can be interpreted as the instantaneous rates of change of the independent variable.
		LO 2.3B: Solve problems involving the slope of a tangent line.	EK 2.3B1: The derivative at a point is the slope of the line tangent to the graph at that point on the graph.
			EK 2.3B2: The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
		LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion.	EK 2.3C1: The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
			EK 2.3C2: The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose known rates are known.
			EK 2.3C3: The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
		LO 2.3D: Solve problems involving rates of change in applied contexts.	EK 2.3D1: The derivative can be used to express information about rates of change in applied contexts.
		LO 2.3E: Verify solutions to differential equations.	EK 2.3E1: Solutions to differential equations are functions or families of functions.
			EK 2.3E2: Derivatives can be used to verify that a function is a solution to a given differential equation.
		LO 2.3F: Estimate solutions to differential equations.	EK 2.3F1: Slope fields provide visual clues to the behavior of solutions of first order differential equations.
	EU 2.4: The Mean Value Theorem	LO 2.4A: Apply the Mean Value Theorem to describe the behavior of a function over an interval.	EK 2.4A1: If a function is continuous over the interval $[a,b]$ and differentiable over the interval (a,b), the Mean Value Theorem guarantees a point within the open interval where the instantaneous rate of change equals the average rate of change over the interval.

Big Idea 3: Integrals and the Fundamental Theorem of Calculus.

	Enduring Understandings (Students will understand that…)	Learning Objectives (Students will be able to…)	Essential Knowledge Statements (Students will know that…)
Big Idea 3: Integrals and the Fundamental Theorem of Calculus	EU 3.1: Antidifferentiation is the inverse process of differentiation.	LO 3.1A: Recognize the antiderivatives of basic functions.	EK 3.1A1: An antiderivative of a function f is a function g whose derivative is f.
			EK 3.1A2: Differentiation rules provide the foundation for finding antiderivatives.
	EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.	LO 3.2A(a): Interpret the definite integral as the limit of a Riemann sum. LO 3.2A(b): Express the limit of a Riemann sum in integral notation.	EK 3.2A1: A Riemann sum, which requires the partition of an interval I, is the sum of products, each of which is the value of a function at a point in the subinterval multiplied by the length of that subinterval of the partition.
			EK 3.2A2: The definite integral of a continuous function f over the interval $\left[a,b\right]$ denoted by $\int_a^b f(x)dx$, is the limit of Riemann sums as the widths of subintervals approach 0. That is, $\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f\left(x_i^*\right)\Delta x_i$ where Δx_i^* is a value in the ith subinterval, Δx_i is the width of the ith subinterval, n is the number of subintervals, and $\max \Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(x_i^*\right)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and Δx_i^* is a value in the ith subinterval.
			EK 3.2A3: The information in the definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
		LO 3.2B: Approximate a definite integral.	EK 3.2B1: Definite integrals can be approximated for functions that ar represented graphically, numerically, algebraically, and verbally.
			EK 3.2B2: Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or non-uniform partitions.
		LO 3.2C: Calculate a definite integral using areas and properties of definite integrals.	EK 3.2C1: In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
			EK 3.2C2: Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
			EK 3.2C3: The definition of the definite integral may be extended to functions with removable or jump discontinuities.

Big Idea 3: Integrals and the FTC (continued)

	Enduring Understandings (Students will understand that…)	Learning Objectives (Students will be able to…)	Essential Knowledge Statements (Students will know that…)
Big Idea 3: Integrals and the Fundamental Theorem of Calculus	EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.	LO 3.3A: Analyze functions defined by an integral.	EK 3.3A1: The definite integral can be used to define new functions; for example, $f(x)=\int_0^x e^{-t^2}\,dt$.
			EK 3.3A2: If f is a continuous function on the interval $[a,b]$, then $\frac{d}{dx}\left(\int_a^x f(t)dt\right)=f(x)$ where x is between a and b.
			EK 3.3A3: Graphical, numerical, algebraic, and verbal representations of a function f provide information about the function g defined as $g(x)=\int_a^x f(t)dt$.
		LO 3.3B(a): Calculate antiderivatives. LO 3.3B(b): Evaluate definite integrals.	EK 3.3B1: The function defined by $F(x)=\int_a^x f(t)dt$ is an antiderivative of f.
			EK 3.3B2: If f is a continuous function on the interval $[a,b]$ and F is an antiderivative of f, then $\int_a^b f(x)dx=F(b)-F(a)$.
			EK 3.3B3: The notation $\int f(x)dx=F(x)+C$ means that $F'(x)=f(x)$ and $\int f(x)dx$ is called an indefinite integral of the function f.
			EK 3.3B4: Many functions do not have closed form antiderivatives.
			EK 3.3B5: Techniques for finding antiderivatives include algebraic manipulation such a long division and completing the square, substitution of variables.
	EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.	LO 3.4A: Interpret the meaning of a definite integral within a problem.	EK 3.4A1: A function defined as an integral represents an accumulation of a rate of change.
			EK 3.4A2: The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
			EK 3.4A3: The limit of an approximating Riemann sum can be interpreted as a definite integral.
		LO 3.4B: Apply definite integrals to problems involving the average value of a function.	EK 3.4B1: The average value of a function f over an interval $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.
		LO 3.4C: Apply definite integrals to problems involving motion.	EK 3.4C1: For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time and the definite integral of speed represents the particle's total distance traveled over the interval of time.
		LO 3.4D: Apply definite integrals to problems involving area and volume.	EK 3.4D1: Areas of certain regions of the plane can be calculated with definite integrals.
			EK 3.4D2: Volumes of solids with known cross sections, including disks and washers, can be calculated with definite integrals.
		LO 3.4E: Use the definite integral to solve problems in various contexts.	EK 3.4E1: The definite integral can be used to express information about accumulation and net change in many contexts.

Big Idea 3: Integrals and the FTC (continued)

	Enduring Understandings (Students will understand that…)	Learning Objectives (Students will be able to …)	Essential Knowledge Statements (Students will know that…)
Big Idea 3: Integrals and the Fundamental Theorem of Calculus	EU 3.5: Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change.	LO 3.5A: Analyze differential equations to obtain general and specific solutions.	EK 3.5A1: Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay.
			EK 3.5A2: Some differential equations can be solved by separation of variables.
			EK 3.5A3: Solutions to differential equations may be subject to domain restrictions.
			EK 3.5A4: The function F defined by $F(x) = c + \int_a^x f(x)dx$ is a general solution to the differential equation $\frac{dy}{dx} = f(x)$, and $F(x) = y_0 + \int_a^x f(x)dx$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$ satisfying $F(a) = y_0$.
		LO 3.5B: Interpret, create, and solve, differential equations from problems in context.	EK 3.5B1: The model for exponential growth and decay that arises from the statement, "The rate of change of a quantity is proportional to the size of the quantity," is $\frac{dy}{dx} = ky$.